

RETRACTS AND Q-INDEPENDENCE

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Dedicated to the memory of Professor Kazimierz Głazek

Abstract

A non-empty set X of a carrier A of an algebra \mathbf{A} is called *Q-independent* if the equality of two term functions f and g of the algebra \mathbf{A} on any finite system of elements a_1, a_2, \dots, a_n of X implies $f(p(a_1), p(a_2), \dots, p(a_n)) = g(p(a_1), p(a_2), \dots, p(a_n))$ for any mapping $p \in Q$. An algebra \mathbf{B} is a *retract* of \mathbf{A} if \mathbf{B} is the image of a *retraction* (i.e. of an idempotent endomorphism of \mathbf{B}). We investigate *Q*-independent subsets of algebras which have a retraction in their set of term functions.

Keywords: general algebra, term function, *Q*-independence, M , I , S , S_0 , A_1 , *G*-independence, *t*-independence, retraction, retract, Stone algebra, skeleton and set of dense element of Stone algebra, Glivenko congruence.

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