Discussiones Mathematicae General Algebra and Applications 27 (2007) 187–197 doi:10.7151/dmgaa.1126



## ON COALGEBRAS AND TYPE TRANSFORMATIONS

H. Peter Gumm

Philipps-University Marburg Faculty of Mathematics and Computer Science Mehrzweckgeb. Hans-Meerwein-Straße 35032 Marburg, Germany

## Abstract

We show that for an arbitrary Set-endofunctor T the generalized membership function given by a sub-cartesian transformation  $\mu$  from T to the filter functor  $\mathbb{F}$  can be alternatively defined by the collection of subcoalgebras of constant T-coalgebras. Sub-natural transformations  $\varepsilon$  between any two functors S and T are shown to be sub-cartesian if and only if they respect  $\mu$ . The class of T-coalgebras whose structure map factors through  $\varepsilon$  is shown to be a covariety if  $\varepsilon$  is a natural and sub-cartesian mono-transformation.

**Keywords:** coalgebra, endofunctor, filter functor, cartesian transformation, crisp.

## 2000 Mathematics Subject Classification: 68Q85, 18C10, 68Q10.

## References

- P. Aczel and N. Mendler, A final coalgebra theorem, pp. 357–365 in: D.H. Pitt et al., eds, Proceedings Category Theory and Computer Science, Lecture Notes in Computer Science, Springer 1989.
- [2] S. Awodey, Category Theory, Oxford University Press (2006).
- [3] H.P. Gumm, Birkhoff's variety theorem for coalgebras, Contributions to General Algebra 13 (2000), 159–173.
- [4] H.P. Gumm, Functors for coalgebras, Algebra Universalis 45 (2001), 135–147.
- [5] H.P. Gumm, From T-coalgebras to filter structures and transition systems, pp. 194–212 in: D.H. Fiadeiro et al., eds, Algebra and Coalgebra in Computer Science, vol 3629 of Lecture Notes in Computer Science, Springer 2005.

- [6] H.P. Gumm and T. Schröder, *Coalgebras of bounded type*, Math. Struct. in Comp. Science **12** (2001), 565–578.
- [7] H.P. Gumm and T. Schröder, *Types and coalgebraic structure*, Algebra Universalis 53 (2005), 229–252.
- [8] E.G. Manes, Implementing collection classes with monads, Math. Struct. in Comp. Science 8 (1998), 231–276.
- [9] J.J.M.M. Rutten, Universal coalgebra: a theory of systems, Theoretical Computer Science 249 (2000), 3–80.
- [10] J.D.H. Smith, Permutation representations of left quasigroups, Algebra Universalis 55 (2006), 387–406.

Received 1 May 2006 Revised 25 July 2006