Discussiones Mathematicae
General Algebra and Applications 27 (2007) 123–139
doi:10.7151/dmgaa.1123

LATTICES OF RELATIVE COLOUR-FAMILIES AND ANTIVARIETIES

Aleksandr Kravchenko

Sobolev Institute of Mathematics SB RAS
Novosibirsk, Russia

e-mail: tclab@math.nsc.ru

Abstract

We consider general properties of lattices of relative colour-families and antivarieties. Several results generalise the corresponding assertions about colour-families of undirected loopless graphs, see [1]. Conditions are indicated under which relative colour-families form a lattice. We prove that such a lattice is distributive. In the class of lattices of antivarieties of relation structures of finite signature, we distinguish the most complicated (universal) objects. Meet decompositions in lattices of colour-families are considered. A criterion is found for existence of irredundant meet decompositions. A connection is found between meet decompositions and bases for anti-identities.

Keywords: colour-family, antivariety, lattice of antivarieties, meet decomposition, basis for anti-identities.

2000 Mathematics Subject Classification: 08C15.

References

[1] V.A. Gorbunov and A.V. Kravchenko, *Universal Horn classes and colour-families of graphs*, Algebra Universalis **46** (1–2) (2001), 43–67.

[2] V.A. Gorbunov and A. V. Kravchenko, *Universal Horn classes and antivarieties of algebraic systems*, Algebra Logic **39** (1) (2000), 1–11.

The work was partially supported by the INTAS (grant 03-51-4110) and the Russian Council for Support of Leading Scientific Schools (grant NSh-4413.2006.1).

[3] L. Lovász, *Operations with structures*, Acta Math. Acad. Sci. Hung. **18** (3–4) (1967), 321–328.

[4] D. Duffus and N. Sauer, *Lattices arising in categorial investigations of Hedetniemi's conjecture*, Discrete Math. **152** (1996), 125–139.

[5] J. Nešetřil and C. Tardif, *Duality theorems for finite structures* (*charaterising gaps and good characterisations*), J. Combin. Theory, B **80** (1) (2000), 80–97.

[6] R. Balbes and Ph. Dwinger, *Distributive lattices*, Univ. Missouri Press, Columbia, MI, 1974.

[7] G. Grätzer, *General Lattice Theory*, Birkhäusser, Basel 1998.

[8] A. Pultr and V. Trnková, *Combinatorial, algebraic and topological representations of groups, semigroups and categories*, Academia, Prague 1980.

[9] Z. Hedrlín, *On universal partly ordered sets and classes*, J. Algebra **11** (4) (1969), 503–509.

[10] J. Nešetřil, *Aspects of structural combinatorics* (*Graph homomorphisms and their use*), Taiwanese J. Math. **3** (4) (1999), 381–423.

[11] J. Nešetřil and A. Pultr, *On classes of relations and graphs determined by subobjects and factorobjects*, Discrete Math. **22** (3) (1978), 287–300.

[12] A.V. Kravchenko, *On lattice complexity of quasivarieties of graphs and endographs*, Algebra and Logic **36** (3) (1997), 164–168.

[13] V.A. Gorbunov and A.V. Kravchenko, *Universal Horn logic, colour-families and formal languages*, General Algebra and Applications in Discrete Mathematics (Proc. Conf. General Algebra Discrete Math.), Shaker Verlag, Aachen, 1997, pp. 77–91.

[14] D.P. Smith, *Meet-irreducible elements in implicative lattices*, Proc. Amer. Math. Soc. **34** (1) (1972), 57–62.

[15] S.S. Goncharov, *Countable Boolean Algebras and Decidability*, Plenum, New York–London–Moscow 1997.

[16] H. Rasiowa and R. Sikorski, *The mathematics of metamathematics*, Panstwowe Wydawnictwo Naukowe, Waszawa 1963.

[17] C. Tardif and X. Zhu, *The level of nonmultiplicativity of graphs*, Discrete Math. **244** (2002), 461–471.

Received 25 April 2006
Revised 21 Juny 2006