Discussiones Mathematicae
General Algebra and Applications 27 (2007) 69–85
doi:10.7151/dmgaa.1120

MAXIMAL SUBMONOIDS OF MONOIDS OF HYPERSUBSTITUTIONS

Ilinka Dimitrova and Jörg Koppitz

University of Potsdam
Institute of Mathematics
Am Neuen Palais, 14415 Potsdam, Germany
e-mail: ilinkad@abv.bg; koppitz@rz.uni-potsdam.de

Abstract

For a monoid M of hypersubstitutions, the collection of all M-solid varieties forms a complete sublattice of the lattice $\mathcal{L}(\tau)$ of all varieties of a given type τ. Therefore, by the study of monoids of hypersubstitutions one can get more insight into the structure of the lattice $\mathcal{L}(\tau)$. In particular, monoids of hypersubstitutions were studied in [9] as well as in [5]. We will give a complete characterization of all maximal submonoids of the monoid $Reg(n)$ of all regular hypersubstitutions of type $\tau = (n)$ (introduced in [4]). The concept of a transformation hypersubstitution, introduced in [1], gives a relationship between monoids of hypersubstitutions and transformation semigroups. In the present paper, we apply the recent results about transformation semigroups by I. Guydzenov and I. Dimitrova ([11], [12]) to describe monoids of transformation hypersubstitutions.

Keywords: regular hypersubstitutions, maximal monoids of hypersubstitutions, transformation semigroups.

2000 Mathematics Subject Classification: 08B05, 08B15, 20M07, 16Y60.

References

[1] V. Budd, K. Denecke and S.L. Wismath, *Short Solid Superassociative Type (n) Varieties*, East-West Journal of Mathematics **2** (2) (2001), 129–145.

[2] Th. Changphas, Monoids of Hypersubstitutions, Dissertation, Universität Potsdam 2004.

[3] Th. Changphas and K. Denecke, *Full Hypersubstitutions and Full Solid Varieties of Semigroups*, East-West Journal of Mathematics **4** (1) (2002), 177–193.

[4] K. Denecke and J. Koppitz, *M-Solid Varieties of Semigroups*, Discuss. Math. **15** (1995), 23–41.

[5] K. Denecke and J. Koppitz, *M*-Solid Varieties of Algebras, Springer Science-Business Media 2006.

[6] K. Denecke, J. Koppitz and S. Niwczyk, *Equational theories generated by hypersubstitutions of type* (n), Int. Journal of Algebra and Computation **12** (6) (2002), 867–876.

[7] K. Denecke, J. Koppitz and S. Shtrakov, *The Depth of a Hypersubstitution*, Journal of Automata, Languages and Combinatorics **6** (3) (2001), 253–262.

[8] K. Denecke and M. Reichel, *Monoids of hypersubstitutions and M-solid varieties*, Contributions to General Algebra 9, Wien 1995, 117–126.

[9] K. Denecke and S.L. Wismath, Hyperidentities and clones, Gordon and Breach Scientific Publishers, 2000.

[10] K. Denecke and S.L. Wismath, *Complexity of Terms, Composition, and Hypersubstitution*, Int. Journal of Mathematics and Mathematical Sciences **15** (2003), 959–969.

[11] V.H. Fernandes, G.M.S. Gomes and M.M. Jesus, *Presentations for Some Monoids of Partial Transformations on a Finite Chain*, Communications in Algebra **33** (2005), 587–604.

[12] Il. Gyudzhenov and Il. Dimitrova, *On the Maximal Subsemigroups of the Semigroup of All Isotone Transformations with Defect* ≥ 2, Comptes rendus de l'Academie bulgare des Sciences **59** (3) (2006), 239–244.

[13] Il. Gyudzhenov and Il. Dimitrova, *On the Maximal Subsemigroups of the Semigroup of all Monotone Transformations*, Discuss. Math., submitted.

[14] J.M. Howie, An Introduction to Semigroup Theory, Academic Press, London 1976.

[15] M.W. Liebeck, C.E. Praeger and J. Saxl, *A Classification of the Maximal Subgroups of the Finite Alternating and Symmetric Groups*, Journal of Algebra **111** (1987), 365–383.

[16] X. Yang, *A Classification of Maximal Subsemigroups of Finite Order-Preserving Transformation Semigroups*, Communications in Algebra **28** (3) (2000), 1503–1513.

Received 23 March 2006
Revised 13 April 2006