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THE DIMENSION OF A VARIETY

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Abstract

Derived varieties were invented by P. Cohn in [4]. *Derived varieties of a given type* were invented by the authors in [10]. In the paper we deal with the derived variety V_σ of a given variety, by a fixed hypersubstitution σ. We introduce the notion of the *dimension of a variety* as the cardinality κ of the set of all proper derived varieties of V included in V.

We examine dimensions of some varieties in the lattice of all varieties of a given type τ. Dimensions of varieties of lattices and all subvarieties of regular bands are determined.

Keywords: derived algebras, derived varieties, the dimension of a variety.

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