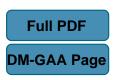
Discussiones Mathematicae General Algebra and Applications 27 (2007) 35–47 doi:10.7151/dmgaa.1117



## THE DIMENSION OF A VARIETY

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## Abstract

Derived varieties were invented by P. Cohn in [4]. Derived varieties of a given type were invented by the authors in [10]. In the paper we deal with the derived variety  $V_{\sigma}$  of a given variety, by a fixed hypersubstitution  $\sigma$ . We introduce the notion of the dimension of a variety as the cardinality  $\kappa$  of the set of all proper derived varieties of V included in V.

We examine dimensions of some varieties in the lattice of all varieties of a given type  $\tau$ . Dimensions of varieties of lattices and all subvarieties of regular bands are determined.

**Keywords:** derived algebras, derived varieties, the dimension of a variety.

**2000 Mathematics Subject Classification:** Primary: 08B99, 08A40; Secondary: 08B05, 08B15.

The results of this paper were presented during the Workshop AAA71 and CYA21 in Będlewo, Poland on February 11, 2006.

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Received 23 February 2006 Revised 10 June 2006